AERODYNAMICS INVESTIGATION OF FINITE WINGS BY LIFTING-LINE MODELS USING THE TRACK METHOD

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ABSTRACT: The current study presents a numerical analysis of the wing lift coefficient including an assessment of geometric effects such as aspect ratio, sweep angle and taper ratio. The circulation is measured through the application of two models which employ the lift line theory. The first model is the classical Prandtl model for zero sweep wing planforms and the second model employs the Weissinger theory of lift line for swept wings. The results demonstrate that for unswept wings the adopted model provides accurate solutions compared to the available literature data. In this context, the lift coefficient grows proportional to the aspect ratio. On the other hand, it decreases with the taper ratio and the sweep angle. These observations occur for all values of aspect ratio.
KEYWORDS: Finite wing, lift line, numerical analysis.

INVESTIGAÇÃO AERODINÂMICA DE ASAS FINITAS POR MODELOS DE LINHA DE SUSTENTAÇÃO UTILIZANDO O MÉTODO DAS FAIXAS

RESUMO: O presente estudo visa analisar numericamente como o coeficiente aerodinâmico de sustentação é influenciado pelos parâmetros geométricos alongamento, afilamento e enflechamento. A circulação é mensurada através da discretização de dois modelos da teoria da linha de sustentação, sendo um o modelo clássico de Prandtl, para asas com enflechamento nulo, e o outro, resultante da teoria da linha de sustentação estendida de Weissinger, para asas enflechadas. Os resultados obtidos nesta análise demonstraram que, para asas com enflechamento nulo, o modelo adotado apresenta resultados coerentes com a literatura e que, para tal disposição tridimensional, o coeficiente de sustentação cresce acentuadamente à medida que se aumenta o alongamento e decresce conforme aumenta o afilamento, especialmente para casos com alongamentos elevados. Verificou-se que o coeficiente de sustentação tem maior sensibilidade com relação ao enflechamento do que em relação ao afilamento. Isto ocorre para todos os valores de alongamento.
PALAVRAS-CHAVE: Asa finita, linha de sustentação, análise numérica.

INTRODUTION

With continuous increase in the prices of fossil fuels, every day, studies focused on the aerodynamics are conveniently found. Research models for flow calculation around aerodynamic surfaces (airfoils and finite wings) have grown exponentially in recent years, this can be credited to the application of these systems in many fields of engineering, for example, land vehicles, turbo machinery and aircraft.

The main applications of Aerodynamics, we can mention the aircraft design, as many aspects studied to determine the best aerodynamic configuration of the aircraft will be widely used for a better analysis of performance and stability as well as for structural calculation, since there are many compromise solutions between a good aerodynamic design and an aircraft with satisfactory performance (Rodrigues, 2014).

The aerodynamic data of a finite wing or an infinite section of this (airfoil) can be analyzed in three ways: experimental, analytical and computational methods. Due to the advent of digital computers, which implies in cost reduction and time optimization in preliminary analyzes, Computational Fluid Dynamics (CFD) methods have become important tools in modern aerodynamics, however, analytical methods can aid in the design of the aerodynamic design, since from the mathematical modeling it is possible to improve the influence of the variables involved.

In this context, the present work seeks to propose a numerical model resulting from the mathematical modeling of the theory of the support line capable of analyzing the influence of the geometric parameters stretching, tapering and sweep back in the aerodynamic coefficient of support.

MATHEMATICAL MODELING

From Fig. (1) it can be visualized the geometric characteristics of a finite wing, where *S* represents the wing area, *b* its wingspan, c_t rope of tip, c_r the root rope, and ϕ o the angle of sweep back. As can be observed, the wing is linearly tapered where the tapering ratio, λ, is defined according to Anderson (2007) by Eq. (1). Another important geometric parameter in finite wing analysis is the elongation, *AR*, mathematically expressed by Eq. (2).

$$\lambda = \frac{c_t}{c_r} \tag{1}$$

$$AR = \frac{b^2}{S} \tag{2}$$

For finite wings, the physical mechanism of lift generation can be explained by the presence of a pressure variation between the upper and lower parts of the wing. This gradient decreases towards the tips (Anderson, 2007). In this way, the current lines that converge behind the wing have different directions, giving rise to two vortices called wing-tip vortices. This deflection of the current lines induces a field of downward velocities known as downwash and this phenomenon modifies the aerodynamic force acting on the wing, producing the so-called induced drag. In an infinite section of the wing, the downwash phenomenon can be represented according to Fig. (2).

Figure 1. Illustration of a finite wing.

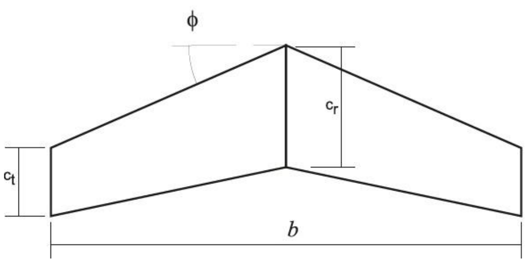
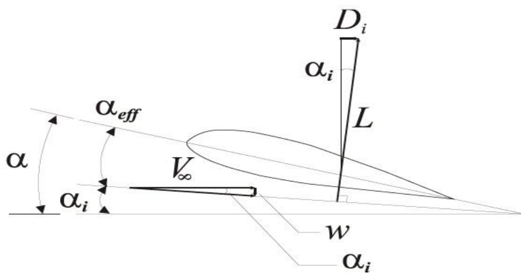


Figure 2. Downwash phenomenon.



From figure 2 it is seen that the downward velocity reduces the angle of attack by a small angle α_i known as the angle of attack induced, this is expressed according to Brederode (2014) by Eq. (3).

$$\alpha_i = \alpha - \alpha_{eff} \tag{3}$$

Considering the flow as incompressible and non-viscous, where the Vortex differential line satisfies the Kutta-Joukowski and Helmholtz theorems, the downwash is calculated according to Bertin and Cummings (2009), by Eq. (4)

$$dw = \frac{\frac{d\Gamma}{dy}dy}{4\pi(y_0 - y)} \tag{4}$$

From Fig. (2) it is observed that induced angle of attack is given by (w/V_∞) where w can be determined by integrating the contributions of the vortex differential elements. Thus α_i is expressed by:

$$\alpha_i = \frac{w}{V_\infty} = \frac{1}{4\pi V_\infty} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\frac{d\Gamma}{dy}}{y_0 - y} dy \tag{5}$$

With respect to the effective angle of attack, if the Kutta-Joukowski theorem is used for the support, it can be modeled mathematically by Eq. (6).

$$\alpha_{eff} = \frac{2\Gamma}{C_l V_\infty c} + \alpha_{L=0} \tag{6}$$

Where C_l is the airfoil lift coefficient, c is the string of this, V_∞ is velocity of the fluid and $\alpha_{(L=0)}$ is the null lift angle of attack By replacing Eqs. (5) and (6) in Eq. (3), we derive the fundamental equation of Prandtl's line of support:

$$\alpha = \frac{2\Gamma}{C_l V_\infty c} + \alpha_{L=0} + \frac{1}{4\pi V_\infty} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\frac{d\Gamma}{dy}}{y_0 - y} dy \tag{7}$$

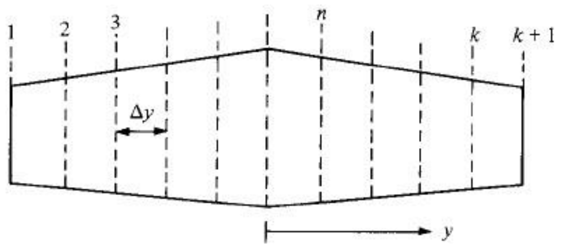
This equation describes the angle of attack as a function of the distribution of circulation around the wingspan. Thus, when the circulation is known, it is possible to calculate the lift by Eq. (8).

$$C_L = \frac{L}{q_\infty S} = \frac{2}{V_\infty S} \int_{-\frac{b}{2}}^{\frac{b}{2}} \Gamma(y)\, dy \tag{8}$$

NUMERICAL MODELING

As seen in the previous section, it is necessary to know the distribution of circulation to determine the coefficient of lift. However, in the analysis of a problem, the circulation distribution is an unknown variable, which can be calculated by writing Eq. (7) discretely for each section of the wing as presented in (Anderson, 2007). Assuming a straight wing, where the experimental data from the airfoil are known, the circulation distribution is iteratively calculated by dividing the wing into several stations using the band method and, for each of these, we can assume values of Γ_1, Γ_2..., Γ_n..., Γ_{k+1} as shown in Fig. (3).

Figure 3. Wing division stations for numerical solution.



Thus, with the variation of Γ it is possible to calculate the induced angle of attack for each station n of the wing.

$$\alpha_i(y_n) = \frac{1}{4\pi V_\infty} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\frac{d\Gamma}{dy}}{y_n - y} dy \tag{9}$$

Using the Simpson integration rule, the integral is written numerically by Eq. (10), where y is the distance between the stations and the indices i and j represent, respectively, a section of the wing and a horseshoe vortex, in which this, of according to the second Helmholtz theorem, is the sum between the two free vortices with the starting vortex.

$$\alpha_i(y_n) = \frac{1}{4\pi V_\infty}\frac{\Delta y}{3}\sum_{j=2,4,6}^{k}\frac{\frac{d\Gamma}{dy}_{j-1}}{y_n - y_{j-1}} + 4\frac{\frac{d\Gamma}{dy}_{j}}{y_n - y_j} + \frac{\frac{d\Gamma}{dy}_{j+1}}{y_n - y_{j+1}} \tag{10}$$

After finding the induced angle of attack, it is possible to calculate the effective angle of attack and, subsequently, the distribution of circulation through the Kutta-Joukowski theorem.

Another model used is the Weissinger extended support line, which is applied to sweeped wings and presented in detail in (Weissinger, 1947). The modeling of this is very similar to the classic theory of Prandtl, except for the boundary conditions. In this model, the circulation distribution is calculated so that the velocity tangent to ¾ of the string is zero, as proposed by the Pistolesi theorem (Wickenheiser, 2007). Thus, this model from the physical point of view is coherent since the flow on a wing must be tangent to the airfoil (for a non-viscous formulation, where we use the condition of slip contour on the wing), therefore, the method of Weissinger derives from a physical condition that is simplified by the contour condition suggested by the Pistolesi theorem.

The computational algorithm for calculating the circulation distribution and the wing lift coefficient follows the sequence described below. The numerical program, whose programming language was the MATLAB® begins with the definition of the geometric parameters, namely: stretching, sweep back, tapering, angle of attack, wingspan and root chord. Then, the induced angle of attack as well as the effective angle of attack, given by Eqs. (10) and (3), is calculated.

Subsequently, the system of equations describing the circulation distribution is calculated and then the coefficient of support is found. If the convergence criterion adopted for the calculation of the circulation distribution, whose value was 10^{-5}, is reached within the time step, it is incremented and a new station is calculated. This sequence is repeated until the maximum number of iterations is reached. If, on the other hand, the convergence criterion is not reached, in the same time step a new input is determined for the induced angle of attack until it reaches convergence.

RESULTS AND DISCUSSION

From the numerical treatment applied to the governing equations of the mathematical model presented previously, it was possible to obtain results for the problem already mentioned. This part of the paper seeks to validate the proposed numerical model, by comparing it with the experimental work presented by (Schlichting, 1979), and then presenting the results obtained with the present numerical model.

Fig. (4) shows the curve of the wing lift coefficient as a function of the elongation, in which a wing with null sweep back was adopted. As can be seen at first glance, there is good agreement with regard to the numerical results presented with the experimental data. It is also verified that the lift coefficient increases as the elongation also increases. In theory, this result was expected because, according to Anderson (2007), for a wing with infinite elongation, the coefficient of lift tends to be the same as that of the airfoilThis is due to the fact that at high elongation values, the three-dimensional effects of the flow are minimized, so that the downwash tends to zero, and consequently the effective and geometric angles of attack are coincident. Still considering null sweep back, it is observed from Fig. (5) that the lift suffers a slight decrease when increasing the wing's tapering, this is most evident at high elongation values. This result is explained by the distribution of circulation between sharp and elliptical wings, which makes the wings with high values of sharpening less efficient in the production of support.

Considering weighted wings, where a value of ϕ=40° was used, it can be observed from Fig. (6) that the firing caused a greater variation of the coefficient of sustentation in relation to the tapering. This occurs for all elongation values.

When analyzed as the flexing influences the lift coefficient, it was observed that the lift remains almost constant for practically all elongation values, up to about 35°. From this value occurs a decrease in the coefficient of sustentation, as presented by Fig. (7).

Figure 4. Validation of results.

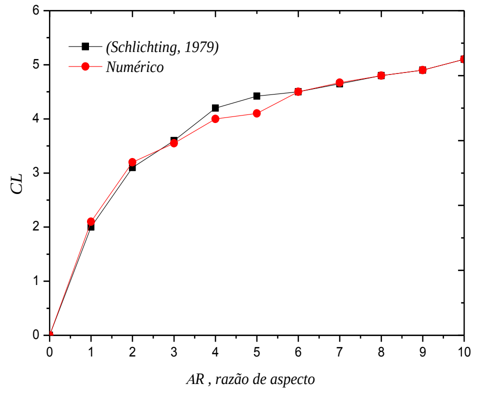


Figura 5. Lifting coefficient,Φ=0°.

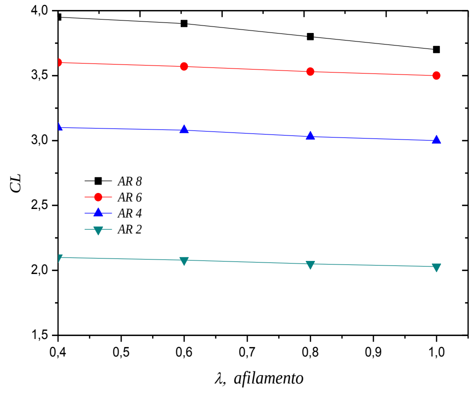


Figure 6. Lifting coeficiente, Φ=40°.

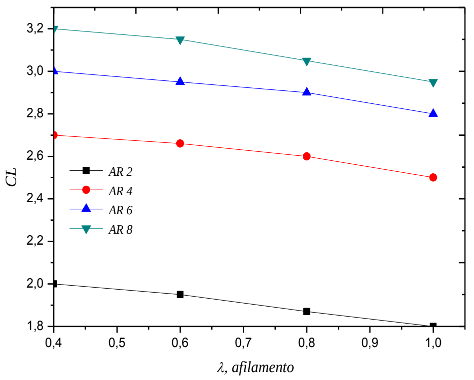
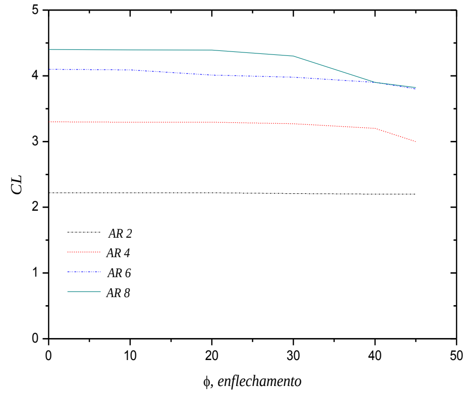


Figure 7. Support x Sweep Back.



CONCLUSION

The results of this work were reached with the explanation of the calculation methods related to the areas of numerical simulation and aerodynamics. With respect to the simulations, it can be observed that the coefficient of support of finite wings grows prominently as the elongation for wings with null sweep back increases. However, such an aerodynamic coefficient decreases smoothly when the grinding is increased. It was also found that for wings with sweep back equal to 40°, the support decreases more sharply than wings with null sweep back.

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